Engineering Notes

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Evaluation of Linearly Thickness-Tapered Booms for Some Space Applications

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Nomenclature

Ax, Ao	= transverse accelerations
d	= mean diameter of the boom
E	= Young's modulus
e	= coefficient of thermal expansion
J	= solar constant
k	= thermal conductivity
S_1, S_2	= maximum values of bending stresses expressed as % values of uniform boom of thickness t_1
t_0, t_1	= wall thickness at the tip and root of the tapered boom
T	= thickness parameter = $(t_1/t_0) - 1$
W	= weight of a linearly thickness-tapered boom expressed as % weight of a uniform boom of thickness t_1
α	= solar absorptance
$\delta_1, \delta_2, \delta_t$	= tip deflections (expressed as % values of uniform boom of thickness t_1)
μ	= mass ratio = ratio of the mass supported at the tip and the mass of a uniform boom of thickness t_2

Introduction

COMMONLY used booms for space applications are in the form of STEMS[1] (storable tubular extendible members). Because of their ease of retraction and extension, they are used for several applications including communication antennas, gravity gradient stabilization, solar panel actuators, and instrument carriers etc.[2] On Apollo 15[3] and 16, two 25-ft BI-STEM† booms were used to extend the mass and gamma ray spectrometers. For each application, the spacecraft designers' prime aim is to reduce weight without affecting their performance. Analysis of thickness-tapered booms[4] indicated significant reduction in weight and improvement in mechanical performance when used as spacecraft antennas. However, if these thickness-tapered booms are used as supporting members for instruments or as structural components of subsystems, their application should be critically analyzed from stiffness and strength considerations. The purpose of this investigation is to evaluate linearly thickness-tapered booms (thickness varying linearly from root to tip) for applications where the boom is used to support a mass at the tip, e.g., an instrument carrier (Fig. 1).

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†Proprietary name for a boom made up of two thin-walled open-section tubes nested with seams in opposition.

The evaluation is based on the maximum stresses and deflection experienced by the boom under space environment.

Analysis

In the fully deployed position the loads experienced by a boom are mainly inertial loads due to orbital maneuvers and thermal input due to solar heating. As the diameter to thickness ratio for thin-walled open-section booms is greater than 100, the assumption of constant diameter along the length is justified. Using the simple beam bending theory, the stresses and deflection for such a boom supporting a tip mass have been obtained as follows:

a) Loading due to constant transverse acceleration Ao constant throughout the length

$$S_1 = (100/3)\,[\,(3+T) + 6\mu(1+T)\,]\,/\,[\,(1+T)\,(1+2\mu)\,] \quad (1)$$

As $T\rightarrow\infty$, $S_1 \rightarrow (100/3)(1+6\mu)\;/(1+2\mu)$ and $\mu=\infty$, $S_1\rightarrow$ 100%.

$$\delta_1 = \frac{400}{(3+8\mu)}\left\{\frac{1}{4}+\frac{2}{3T}-\frac{1}{T^2}+\frac{2}{T^3}-\frac{2}{T^4}\log_e(1+T)\right.$$
$$\left.+6\mu\left(1+\frac{1}{T}\right)\left[\frac{1}{2}-\frac{1}{T}+\frac{1}{T^2}\log_e(1+T)\right]\right\} \quad (2)$$

As $T\rightarrow\infty$, $\delta_1\rightarrow 100(1+12\mu)/(3+8\mu)$, and as $T\rightarrow\infty$ and $\mu\rightarrow\infty$, $\delta_1\rightarrow 150\%$.

The maximum bending stress occurs at the root and the maximum deflection occurs at the tip.

b) Loading caused by linearly varying transverse acceleration expressed as $Ax=Ao(1-x/L)$: The following relations have been obtained for the bending stress and tip deflection

$$S_2 = 25[\,(4+T) + 12\mu(1+T)\,]\,/\,[\,(1+T)\,(1+3\mu)\,] \quad (3)$$

As $Tu\rightarrow\infty$, $S_2\rightarrow 25(1+12\mu)/(1+3\mu)$ and $\mu\rightarrow\infty$, $S_2\rightarrow 100\%$.

$$\delta_2 = \frac{1000}{(11+40\mu)}\left\{\frac{3}{10}+\frac{13}{12T}-\frac{5}{3T^2}+\frac{7}{2T^3}+\frac{1}{T^4}\right.$$
$$-\frac{1}{T^4}\left(4+\frac{1}{T}\right)\log_e(1+T)$$
$$\left.+12\mu\left(1+\frac{1}{T}\right)\left[\frac{1}{2}-\frac{1}{T}+\frac{1}{T^2}\log_e(1+T)\right]\right\} \quad (4)$$

As $T\rightarrow\infty$, $\delta_2\rightarrow 100(3+60\mu)/(11+40\mu)$ and as $T\rightarrow\infty$ and $\mu\rightarrow\infty$, $\delta_2\rightarrow 150\%$.

For this case also maximum stress occurs at the root and the maximum deflection at the tip.

c) Thermal bending due to solar heating: The exact analysis of the thermal behavior of the open-section overlapped booms

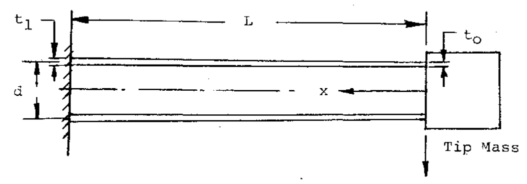

Fig. 1 A linear thickness-tapered boom supporting a tip mass.

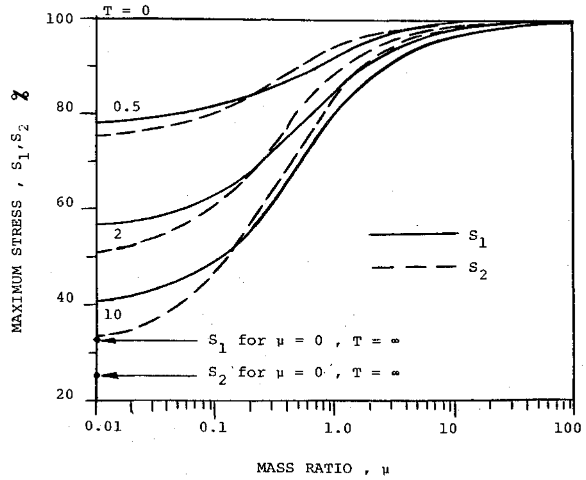

Fig. 2 Maximum bending stress due to inertial loads.

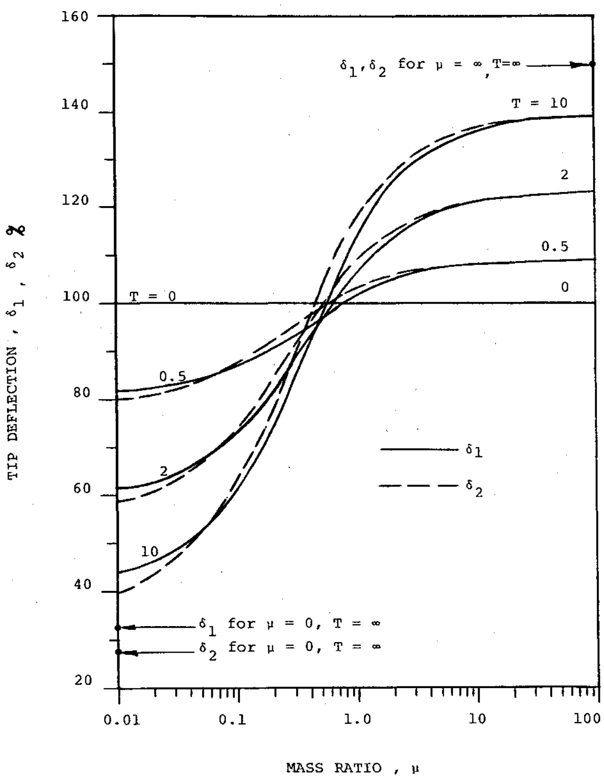

Fig. 3 Tip deflection caused by the inertial loads.

is complicated. However, based on physical considerations[5], the following expression for apparent thermal bending moment was established

$$M = \beta E e d^4 J \alpha / k \tag{5}$$

Where β is a constant depending upon the boom section geometry and its oreitnation with respect to the sun.

If the boom is free to expand and contract, the moment M results in thermal bending and no stresses develop. A restrained thermal expansion of the boom results in stresses but the amount of thermal bending is reduced depending upon the amount of restraint. Considering no restraints, the maximum tip deflection has been obtained by using Eq. (5) and beam bending theory

$$\delta_t = 200(1+T)\,[T - \log_e(1+T)\,]/T^2 \tag{6}$$

As $T \to \infty$, $\delta_t \to 200\%$. It should be noticed that this deflection does not depend upon the tip mass.

d) Weight of the boom: The weight of a linearly thickness-tapered boom as a percentage weight of a uniform boom of thickness t_l can be expressed as

$$W = 50(2+T)/(1+T) \tag{7}$$

As $T \to \infty$, $W \to 50\%$.

Results and Discussion

Equations (1-6) can be utilized to compare the mechanical performance of a linearly thickness-tapered boom with a uniformly thick boom when the application involves supporting a mass at the tip. Figure 2 gives the maximum bending stress experienced by the tapered boom as a percentage of the corresponding stress of a uniformly thick boom under constant and linearly varying acceleration loads. It can be seen that for low-mass ratios these stresses are significantly lower than the values for a uniform boom. Even for high-mass ratios, the stress values never exceed the values for uniform boom for any thickness parameter T. The maximum deflection caused by the inertial loads has been plotted as a function of mass ratio in Fig. 3 for different values of the thickness parameter. The advantage of the thickness-tapered boom in terms of tip deflection can be seen only for mass ratio of less than 0.5. However, even for high-mass ratio and high-thickness parameter ($\mu \to \infty$ and $T \to \infty$), the deflection value is not more than 50% higher than that of a uniform boom. The variation in the maximum thermal deflection as a function of the thickness parameter is given by Eq. (6). The deflection increases as the taperness of the boom is increased. As $T \to \infty$, the predicted value of thermal deflection approaches twice that of uniform boom. The advantage of the thickness-tapered boom in terms of weight reduction is evident from Eq. (7). It can be seen that maximum possible weight saving by using a linearly thickness-tapered boom is 50%.

The previous results indicate that a thickness-tapered boom can be used for supporting tip mass with the advantages of reduced bending stresses and saving in weight if the values of tip deflection are found to be within an allowable limit.

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Graphic Representation of Superorbital Rocket Performance

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THIS note deals with the usefulness of the nuclear-thermal and high-energy chemical rocket drives in the last stage of multistage vehicles. Its purpose is to supplement and

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